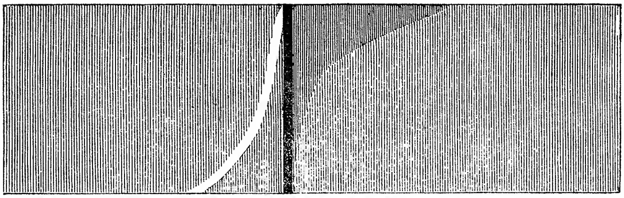

Fig. 7.

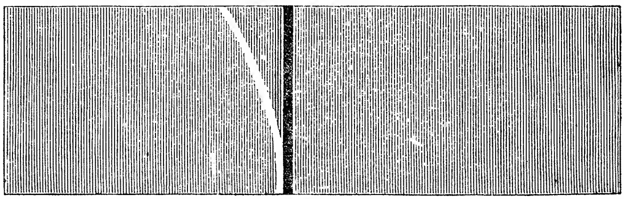

Fig. 8.

8. At times, D puts on the appearance of the limiting line of a channelled-space spectrum, the " easing off " of the absorption being now on one side and now on the other.

9. Should all these phenomena be ultimately referred to the causes which produce a channelled-space spectrum (one of which undoubtedly is the tendency to a unilateral instead of a bilateral widening), a line-spectrum will be regarded as a special case merely, and not as an entirely different spectrum, as it has been hitherto ; and the range of molecular combinations in any one element from which line-spectra may be produced is extended.

10. The question further arises, whether many of the short lines in spectra are not remnants of channelled-space spectra.

June 18, 1874.

JOSEPH DALTON HOOKER, C.B., President, in the Chair.

Mr. Henry Bowman Brady, Mr. Augustus Wollaston Franks, Prof. Olaus Henrici, Sir Henry Sumner Maine, and Mr. Osbert Salvin were admitted into the Society.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read :—

I. " A Contribution to the Anatomy of Connective Tissue, Nerve, and Muscle, with special reference to their connexion with the Lymphatic System." By G. Thin, M.D. Communicated by Prof. Huxley, Sec. R.S. Received April 22, 1874*.

* This Paper will appear in No. 155.

II. "Given the Number of Figures (not exceeding 100) in the Reciprocal of a Prime Number, to determine the Prime itself." By WILLIAM SHANKS. Communicated by the Rev. G. SALMON, F.R.S. Received May 19, 1874.

In a former communication (*suprà*, p. 200) I gave a Table showing the number of figures in the period of the reciprocal of every given prime up to 20,000. The Table here introduced is intended to solve the converse problem, and to show what primes have a given number of figures in their period. It appears at once, from the ordinary rule for converting a pure circulating decimal into a proper fraction, that if the reciprocal of a prime have n figures in its period, that prime must be a factor in the number formed by writing down n nines, and therefore also, generally, in the number formed by writing down n ones. We denote that number by n; that is to say, 5 (in the left column), for example, $=11111$, except where $3, 3^2, 3^3 \ldots . 3^6$ are concerned, when we have 3, for example, $=999$. The problem now before us is equivalent to that of breaking up n into its prime factors; and the previous Table gives us great facility in doing this, for it exhibits every factor of n which is less than 20,000*; and if, after accounting for all these, the remaining factor of n is less than $30{,}000^2$, we may be sure that it is a prime number, and that the resolution is complete.

If we have to deal with a composite number mn, this may obviously be written down either as m groups of n ones or as n groups of m ones. It follows that mn contains m and n as factors. We may also state here that 12, besides the factor 9901, obviously has all the factors belonging to any submultiple of 12, *e.g.* 2, 3, 4, 6; and that this holds in all other similar cases, and need not be stated again. When we affirm that the resolution in any case is complete (and, indeed, throughout the Table), it is to be clearly understood that the submultiples have all been carefully attended to, and thus any result may easily be verified. The high factors found (those, we mean, above $30{,}000^2$) have involved considerable labour; and though we may not say absolutely that they are primes, yet we are certain that, if composite, their component factors are primes each greater than 30,000, and that the periods of their reciprocals have readily been found. It only remains to add here that the left column contains the given number of figures in the reciprocal of the prime or primes found and placed opposite in the right column, or, in a few cases, of the second powers of primes, and as far as the sixth power of the prime 3.

If the number of figures in the reciprocal of P be n, then the general rule†, which may be drawn from particular cases such as the following two, is that the number of figures in the reciprocal of P^2 is nP, of P^3 is

* In point of fact I have carried on the calculation up to 30,000.

† See 'Messenger of Mathematics,' vol. ii. pp. 41–43 (1872), and vol. iii. pp. 52–55 (1873).

$n\mathrm{P}^2$, and so on. Since the period of $\frac{1}{19}=18$, and since the remainders resulting from dividing 18 such periods successively by 19 are, in order, 15, 11, 7, 3, 18, 14, 10, 6, 2, 17, 13, 9, 5, 1, 16, 12, 8, 4, 0, it follows that $\frac{1}{19^2}=18\times 19=342$. The law of such remainders, after the first has been obtained, is simple enough, and may be written down at once. Again, since the period of $\frac{1}{163}=81$, also since the remainders resulting from dividing 163 such periods, each of 81 figures, successively by 163 are, in order, 149, 135, 121, 107, 93, 79, 65, 51, 37, 23, 9, 158, 144. . . . 0 (the series consisting of 163 terms, of which the last is 0), it follows that $\frac{1}{163^2}=81\times 163=13203$. The law of the above series is evident, and the number of terms is easily found to be 163. There is an obvious exception when $\mathrm{P}=3$; then the period is divisible by P, and the number of figures in the reciprocal of 3^2 is 1, of 3^3 is 3, and of 3^n is 3^{n-2}. There are other exceptions also, or at all events one. Desmarest, for instance, has remarked that in the case of $\mathrm{P}=487$, the period is divisible by 487; and therefore the number of figures in the reciprocal of 487^2 is the same as that in the reciprocal of 487, viz. 486. I am not acquainted with the general theory of such exceptions; nor do I know what other primes (if any) besides 3 and 487 have the same peculiarity.

With these explanations the following Table can readily be understood. We mark with an asterisk those cases in which the resolution is complete, thus 28 | 29 . 281 . 12149 9449. We are to be understood as affirming that 12149 9449 is a prime number.

Given number of figures in Period of Primes.	Primes, Prime Factors, &c.
1*	3 and 3^2.
2*	11.
3*	3^3 . 37.
4*	101.
5*	41 . 271.
6*	7 . 13.
7*	239 . 4649.
8*	73 . 137.
9*	3^4 . 333667.
10*	9091.
11*	21649 . 513239.
12*	9901.
13*	53 . 79 . 26537 1653.
14*	90909 1.
15*	31 . 29061 61.
16*	17 . 58823 53.
17	Seems prime.
18*	19 . 52579.
19	Seems prime.
20*	3541 . 27961.

Given number of figures in Period of Primes.	Primes, Prime Factors, &c.
21*	$43 \cdot 1933 \cdot 10838\ 689$.
22*	$11^2 \cdot 23 \cdot 4093 \cdot 8779$.
23	Seems prime.
24*	99990 001.
25	21401 . 25601 . 18252 12130 01.
26	859 . 10583 13049.
27	$3^5 \cdot 757 \cdot 44033\ 46547\ 77631$.
28*	29 . 281 . 12149 9449.
29	3191 . 16763 . 20772 03000 95927 10406 7.
30*	211 . 241 . 2161.
31	2791 . 39810 50201 04303 51526 73275 21.
32*	353 . 449 . 641 . 1409 . 69857.
33	67 . 13446 28210 31329 8373.
34	103 . 4013 . 21993 83336 9.
35	71 . 12676 18436 74776 04353 521.
36	99999 90000 01.
37	Seems prime.
38	90909 09090 90909 091.
39	90090 09009 00990 99099 0991.
40	99990 00099 99000 1.
41	83 . 1231 . 10874 80167 08045 28702 40778 98379 32830 7.
42*	$7^2 \cdot 127 \cdot 2689 \cdot 45969\ 1$.
43	173 . 64226 07578 67694 28387 92549 77520 87347 46307.
44	89 . 11124 70797 64156 1909.
45	99900 00009 99000 99999 9001.
46	47 . 139 . 2531 . 54979 71844 91917.
47	Seems prime.
48	99999 99900 00000 1.
49	10000 00100 00001 00000 01000 00010 00000 10000 001.
50	251 . 5051 . 78875 94347 2201.
51	613 . 14696 58892 17112 70961 00994 95907.
52	521 . 19003 81976 77733 22437 81.
53	107 . 10384 21599 16926 27206 64589 82346 83281 41225 33748 70197 3.
54	99999 99990 00000 001.
55	1321 . 68130 88570 01514 75398 18244 51098 41022 71.
56	7841 . 12752 20010 20150 50376 1.
57	21319 . 42258 12190 53849 10220 50710 59144 89.
58	59 . 15408 32049 30662 55778 12018 49.
59	Seems prime.
60	61 . 16557 36049 01641.
61	733 . 4637 . 32690 11286 55567 78492 67785 60346 38966 63414 98113 99297 3391.
62	90909 09090 90909 09090 90909 09091.
63	10837 . 23311 . 39545 35794 55592 00238 00680 443.
64	19841 . 50400 68544 93221 10780 70661 761.
65	90000 90000 90090 90090 90090 99090 99099 99099 991.
66	10989 01098 89010 98901 1.
67	Seems prime.
68	99009 90099 00990 09900 99009 90099 01.
69	277 . 32523 49822 74693 46602 92093 53758 09022 01840 83.
70	10999 88890 11110 98889 00011.
71	Seems prime.
72	3169 . 31555 69580 30609 02492 9.
73	Seems prime.
74	7253 . 12533 99847 08521 86556 03324 01639 447.
75	151 . 4201 . 15763 98555 37391 91709 16417 09400 63151.
76	99009 90099 00990 09900 99009 90099 00990 1.
77	5237 . 17185 41321 38439 75575 73019 07599 58180 44493 01317 37618 86404 43.
78	$13^2 \cdot 157 \cdot 6397 \cdot 84166\ 49699\ 61183\ 43$.
79	317 . 6163 . 10271 . 55372 39794 64587 20397 50752 71926 68846 36072 32019 52048 12389 25326 15741 471.

Given number of figures in Period of Primes.	Primes, Prime Factors, &c.
80	99999 91000 00000 99999 99100 00000 1.
81	3^6 . 163 . 9397 . 21762 15574 17380 51978 03850 29334 29783 20758 07163 797.
82	90909 09090 90909 09090 90909 09090 90909 09091.
83	Seems prime.
84	10099 98990 00099 98990 00101.
85	90000 90000 90000 90900 90900 90900 90909 90909 90909 90909 99909 99909 9991.
86	90909 09090 90909 09090 90909 09090 90909 09090 91.
87	4003 . 22505 64329 00549 81286 55760 43195 08116 66025 25583 29000 997.
88	617 . 16205 83484 60129 67584 92708 26564 02106 953.
89	Seems prime.
90	29611 64229 50923 97453 00732 9.
91	547 . 14197 . 17837 . 64973 58525 58248 78623 76372 29838 67691 22282 27693 73769 82738 03847 7.
92	1289 . 76811 40495 74080 75951 11722 18851 05500 46470 9.
93	90090 09009 00900 90090 09009 00900 99099 09909 90990 99099 09909 90991.
94	6299 . 14432 30527 21211 15905 84364 04046 81839 83027 609.
95	191 . 47120 89005 70681 09952 82727 69638 69115 13141 30942 35654 39842 88481 62827 17801.
96	97 . 10309 27835 05154 62886 59793 81443 3.
97	Seems prime.
98	197 . 50761 41624 36553 29949 18781 72639 59390 35533.
99	199 . 397 . 12657 74717 41579 43369 23914 28173 61378 68182 22092 83191 77752 74356 67.
100	99999 99999 00000 00000 99999 99999 00000 00001.

Note.—In the preparation of this paper valuable assistance was received from the Rev. Prof. Salmon, F.R.S., both in the way of suggestions and otherwise.—W. S.

Houghton-le-Spring,
April 18, 1874.

III. "On the Number of Figures in the Reciprocal of every Prime between 20,000 and 30,000." By William Shanks. Communicated by the Rev. George Salmon, F.R.S. Received June 6, 1874.

In a former communication* I gave the number of figures in the reciprocal of every prime below 20,000; the present Table is simply an extension of the former, and has been calculated by the same method. Towards the close of the former Table, viz. opposite the prime 19841, *instead* of 1984 *read* 64. The *whole* of the former Table has kindly been verified by the Rev. Dr. Salmon. For the accuracy of the following Table I am entirely responsible, and believe it is free from error.

* *Suprà*, p. 200.